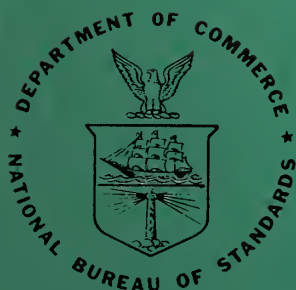




TECHNICAL NOTE

435

The Hyperbolic Character of Certain Experimental Results Which Tend Toward Limiting Values



U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards

THE NATIONAL BUREAU OF STANDARDS

The National Bureau of Standards¹ provides measurement and technical information services essential to the efficiency and effectiveness of the work of the Nation's scientists and engineers. The Bureau serves also as a focal point in the Federal Government for assuring maximum application of the physical and engineering sciences to the advancement of technology in industry and commerce. To accomplish this mission, the Bureau is organized into three institutes covering broad program areas of research and services:

THE INSTITUTE FOR BASIC STANDARDS . . . provides the central basis within the United States for a complete and consistent system of physical measurements, coordinates that system with the measurement systems of other nations, and furnishes essential services leading to accurate and uniform physical measurements throughout the Nation's scientific community, industry, and commerce. This Institute comprises a series of divisions, each serving a classical subject matter area:

—Applied Mathematics—Electricity—Metrology—Mechanics—Heat—Atomic Physics—Physical Chemistry—Radiation Physics—Laboratory Astrophysics²—Radio Standards Laboratory,² which includes Radio Standards Physics and Radio Standards Engineering—Office of Standard Reference Data.

THE INSTITUTE FOR MATERIALS RESEARCH . . . conducts materials research and provides associated materials services including mainly reference materials and data on the properties of materials. Beyond its direct interest to the Nation's scientists and engineers, this Institute yields services which are essential to the advancement of technology in industry and commerce. This Institute is organized primarily by technical fields:

—Analytical Chemistry—Metallurgy—Reactor Radiations—Polymers—Inorganic Materials—Cryogenics²—Office of Standard Reference Materials.

THE INSTITUTE FOR APPLIED TECHNOLOGY . . . provides technical services to promote the use of available technology and to facilitate technological innovation in industry and government. The principal elements of this Institute are:

—Building Research—Electronic Instrumentation—Technical Analysis—Center for Computer Sciences and Technology—Textile and Apparel Technology Center—Office of Weights and Measures—Office of Engineering Standards Services—Office of Invention and Innovation—Office of Vehicle Systems Research—Clearinghouse for Federal Scientific and Technical Information³—Materials Evaluation Laboratory—NBS/GSA Testing Laboratory.

¹ Headquarters and Laboratories at Gaithersburg, Maryland, unless otherwise noted; mailing address Washington, D. C., 20234.

² Located at Boulder, Colorado, 80302.

³ Located at 5285 Port Royal Road, Springfield, Virginia 22151.

UNITED STATES DEPARTMENT OF COMMERCE
Alexander B. Trowbridge, Secretary
NATIONAL BUREAU OF STANDARDS • A. V. Astin, Director



TECHNICAL NOTE 435


ISSUED NOVEMBER 1967

The Hyperbolic Character of Certain Experimental Results Which Tend Toward Limiting Values

Arthur F. Kirstein

Metrology Division
Institute for Basic Standards
National Bureau of Standards
Washington, D.C. 20234

NBS Technical Notes are designed to supplement the Bureau's regular publications program. They provide a means for making available scientific data that are of transient or limited interest. Technical Notes may be listed or referred to in the open literature.



Digitized by the Internet Archive
in 2012 with funding from
LYRASIS Members and Sloan Foundation

<http://www.archive.org/details/hyperboliccharac00kirs>

THE HYPERBOLIC CHARACTER OF CERTAIN EXPERIMENTAL RESULTS WHICH TEND TOWARD LIMITING VALUES

by Arthur F. Kirstein

ABSTRACT

The hyperbolic character of certain experimental data obtained from structural and mechanical testing provides an extremely simple graphical method for use in approximating the limiting values associated with the data. The method, developed by Southwell to explain the behavior of eccentrically loaded thin elastic struts, is shown to have much wider application. A limited number of illustrations are given to describe the method and to show how it might be used as a research tool.

Key words: Experimental, graphical solution, hyperbolic character, limiting values.

1. Introduction

It is a common occurrence in experimental investigations to find that plotted test results may form a curve which passes through the origin and tends toward limiting values in much the same manner as a rectangular hyperbola having horizontal and vertical asymptotes. Experimental data possessing this hyperbolic character will approach limiting values asymptotically. Thus an extremely simple graphical method is provided for use in approximating the limiting values of asymptotes. The equation for the rectangular hyperbola used in this method of approximation was given by Ayrton and Perry^[1]* and later developed by Southwell^[2] to explain the behavior of eccentrically loaded thin elastic struts. This method was also used by Ross^[3] in a slightly modified form to represent the creep of concrete under load.

This investigation explored a number of different instances where the hyperbolic character of experimental data was used to illustrate

*Figures in brackets indicate the literature references at the end of this paper.

the method of approximation as a useful research tool. The basic method was explained and some of the variations or extensions were also shown.

2. Background

To the writer's knowledge the first published account recognizing this hyperbolic character in the load vs deflection data obtained from eccentrically loaded struts was given by Ayrton and Perry. They found that when the reciprocals of deflection and stress from tests of oak struts were plotted on rectangular coordinates the points fell very nearly on a straight line. This was expressed as

$$\delta = \frac{\alpha\sigma}{\beta - \sigma} \quad (1)$$

which can be written in the form used by Southwell nearly one half century later

$$\delta - \beta \frac{\delta}{\sigma} + \alpha = 0 \quad (2)$$

where δ = deflection

σ = stress

α = initial eccentricity

β = critical elastic stress.

When equation (2) is divided by δ we recognize that a linear relationship exists between the reciprocals of σ and δ .

The use of this graphical method of approximating limiting values can be explained more fully by examining the properties of the hyperbola. Consider the rectangular hyperbola passing through the origin of the cartesian coordinate system shown in figure 1 having as asymptotes

$$x + \alpha = 0 \quad (3)$$

and

$$y - \beta = 0;$$

the equation of this hyperbola is

$$xy - \beta x + \alpha y = 0. \quad (4)$$

Equation (4) may be written as

$$x - \beta v + \alpha = 0 \quad (5)$$

where

$$v = \frac{x}{y} . \quad (6)$$

Equation (5) is recognized as the equation for the straight line shown in figure 2. The inverse slope and the intercept on the x axis of this line are measures of β and α , respectively, which are the limiting values or asymptotes of the original hyperbola shown in figure 1.

The application of this method to experimental data is accomplished graphically by plotting the data in the form shown in figure 2. The linearity of the resulting curve is a measure of how well the data approximates the rectangular hyperbola and the limiting values or asymptotes may be determined as described previously.

Southwell found that the load-deflection relationship for an eccentrically loaded thin elastic column approximated this hyperbola and compared the critical load of the column to the predicted limiting value, β . The results of this comparison were very good. No doubt, this along with other factors accounts for the assumption that the method would apply only so long as the system remained elastic.

Conversely, Ross used this method in a slightly modified form to represent the creep of concrete under load. Thus Ross indicated that the method could be used to predict limiting values other than critical elastic loads as supposed by Southwell.

Certain other experimental data which may be observed from mechanical or structural testing may be typified by the curves shown in figure 3. The curves shown in figure 3a were modified as indicated to produce the curves representing stress vs nonlinear strain in figure 3b. This procedure is similar to that used by Galletly and Reynolds^[4] which produces curves that resemble Southwell's rectangular hyperbola of figure 1. The modified results shown in figure 3b can be used in the graphical method of approximating the limiting values of the curves given in figure 3a. The success or failure of this method in predicting the limiting value is not dependent upon elastic action, but is dependent on how well the experimental results approximate the hyperbola.

3. Limiting Values and the Hyperbolic Character of Experimental Results

It was indeed unfortunate that Southwell arbitrarily restricted the use of his method to the examination of the critical elastic buckling loads. Had he continued to examine this method when applied to Kármán's medium and thick struts^[5] he would have found that it would have served to predict with a fair degree of accuracy the ultimate load

carrying capacities of the struts. Table 1 contains the data from these tests of medium and thick struts. Column three of this table contains the values predicted from Southwell's rectangular hyperbola and column four gives the ratio of these predicted limiting values to those observed by Kármán. The good agreement shown in table 1 has little or nothing to do with the elasticity of the medium and thick struts. It merely shows that the load vs deflection curves possess this hyperbolic character which is useful for the purpose of approximating the values of the horizontal asymptotes or limiting values which in this case correspond to the ultimate loads.

Using the previously discussed modification of typical stress vs strain data as it pertains to figures 2 and 3, consider the stress vs strain curves for compressive tests on standard 6 x 12 inch concrete cylinders shown in figure 4. Applying the linearity test which is inherent in the graphical method shown in figure 2 to these data produced the results given in figure 5. Note that the curves in figure 5 are linear for the larger values of nonlinear strain, which indicates that the data does at least approximate the rectangular hyperbola. It should be noted that in the graphical method illustrated the straight line is selected so that it coincides more closely with data associated with the larger values of nonlinear strain, and no attempt was made to apply the method of least squares or other mathematic method of curve fitting. It has been found from experience^[3] that the graphical method produces results which more closely represent the limiting values. A comparison of the limiting values obtained from these curves and the compressive strengths determined from the test results is made in table 2. These results show that the method can be used to make reasonable predictions of the compressive strength.

Ross found that the creep in concrete when plotted as the ordinate against the corresponding time as the abscissa produced a curve similar in appearance to the rectangular hyperbola shown in figure 1. Further, he plotted a curve of time/creep vs time (t/c vs t) which he referred to as the "reduction of creep curve". This curve was similar to that shown in figure 2, however, Ross elected to write the linear equation for the curve as

$$\frac{t}{c} = a + bt \quad (7)$$

where c = creep per unit length in time, t

a = a constant (t/c intercept)

b = a coefficient (slope)

Ross found it useful to correlate a and b of equation (7) with factors or conditions which affected the results of his studies of creep.

4. Summary

It has been observed that the graphical method of predicting the limiting values or asymptotes for experimental data that exhibit this hyperbolic character has a wider application than was recognized in the past. It is obvious from the results presented in table 1 that the usefulness of the method extends beyond that ascribed to it by Southwell when dealing with the stability of struts. Further, it was shown in table 2 that the method has some degree of usefulness in predicting mechanical properties associated with materials testing. Ross also found this method to be of use in predicting the limiting values of creep in concrete.

As noted it is fairly common in experimental investigations involving structural and mechanical testing to obtain test results which tend toward limiting values asymptotically. Furthermore, from the previously cited examples it would appear that the method described could be applied as a very useful tool in the structural or mechanical testing laboratory to predict results from limited investigation and to plan testing programs which would yield a greater amount of useful results with a lesser amount of testing. It should also be recognized that the graphical method can be used to write empirical equations which show the relationship of the variables involved.

5. Acknowledgments

Acknowledgment is made to Mr. R. G. Mathey of the NBS Structures Section for furnishing the data presented in figure 4.

6. References

1. Ayrton, W. E., and Perry, J., "On Struts", The Engineer, Dec. 10, 1886, pp. 464-465 and Dec. 24, 1886, pp. 513-515.
2. Southwell, R. V., "On the Analysis of Experimental Observations in Problems of Elastic Stability", Proc. Royal Society, vol. 135A, pp. 601-616 (London), 1932.
3. Ross, A. D., "Concrete Creep Data", The Structural Engineer, vol. 15, no. 8, pp. 314-326, 1937.
4. Galletly, G. D., and Reynolds, T. E., "A Simple Extension of Southwell's Method for Determining the Elastic General Instability Pressure of Ring-Stiffened Cylinders Subject to External Hydrostatic Pressure", Proc. SESA, vol. XIII, no. 2, 1956.
5. von Kármán T., "Untersuchungen über Knickfestigkeit" (also printed in "Mitteilungen über Forschungsarbeiten", published by the Ver. Deuts.Ing., Berlin, 1909).

Table 1. - Comparison of Kármán's Observed Maximum Stress and the Predicted Limiting Value

Strut No.	Maximum stress		Predicted stress
	Observed	Predicted	Observed stress
	kg/cm ²	kg/cm ²	
Medium struts			
7A	2760	2794	1.012
7B	2685	2796	1.041
8	2740	2724	0.994
9A	3030	3058	1.009
9B*	--	--	--
10A	3185	3255	1.022
10B	3080	3120	1.013
11	3165	3185	1.006
12A*	--	--	--
12B	2960	3009	1.017
13	3060	3105	1.015
Thick struts			
14B	3320	3439	1.036
15A	3395	3440	1.015
15B	3485	3360	0.964
16	3890	3989	1.025
17	4330	4510	1.042

*Insufficient data recorded for determination of predicted value

Table 2.- Observed and Predicted Compressive Strength
of 6 by 12 in. Concrete Cylinders

Cylinder	<u>Compressive strength</u>		<u>Predicted strength</u>
	<u>Observed</u>	<u>Predicted</u>	<u>Observed strength</u>
	(psi)	(psi)	
1	3360	3566	1.061
2	3077	3271	1.063
3	2911	3090	1.061
4	2844	2922	1.027
5	2741	2960	1.080

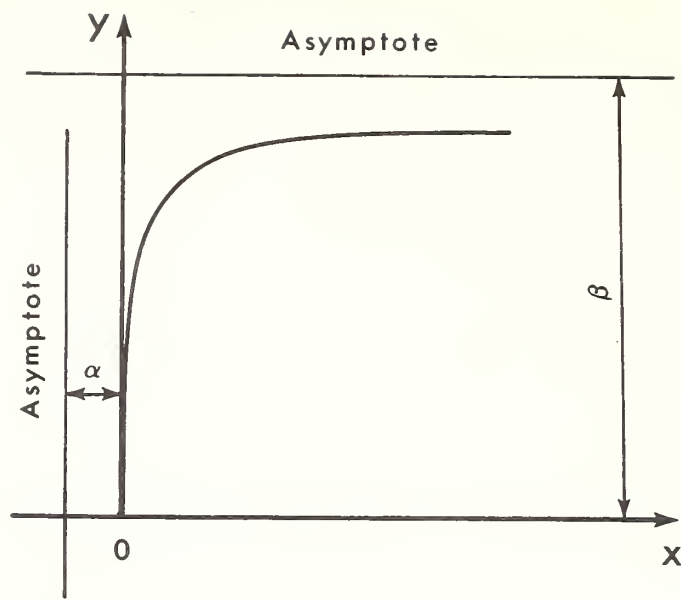


Figure 1. Rectangular hyperbola

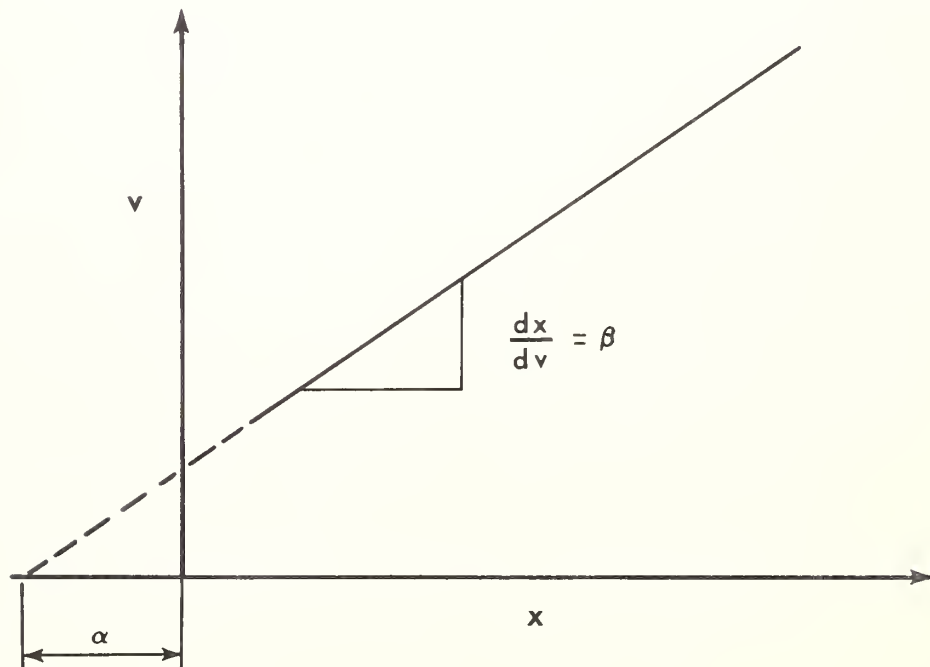


Figure 2. Relationship between v and x of equation (5).

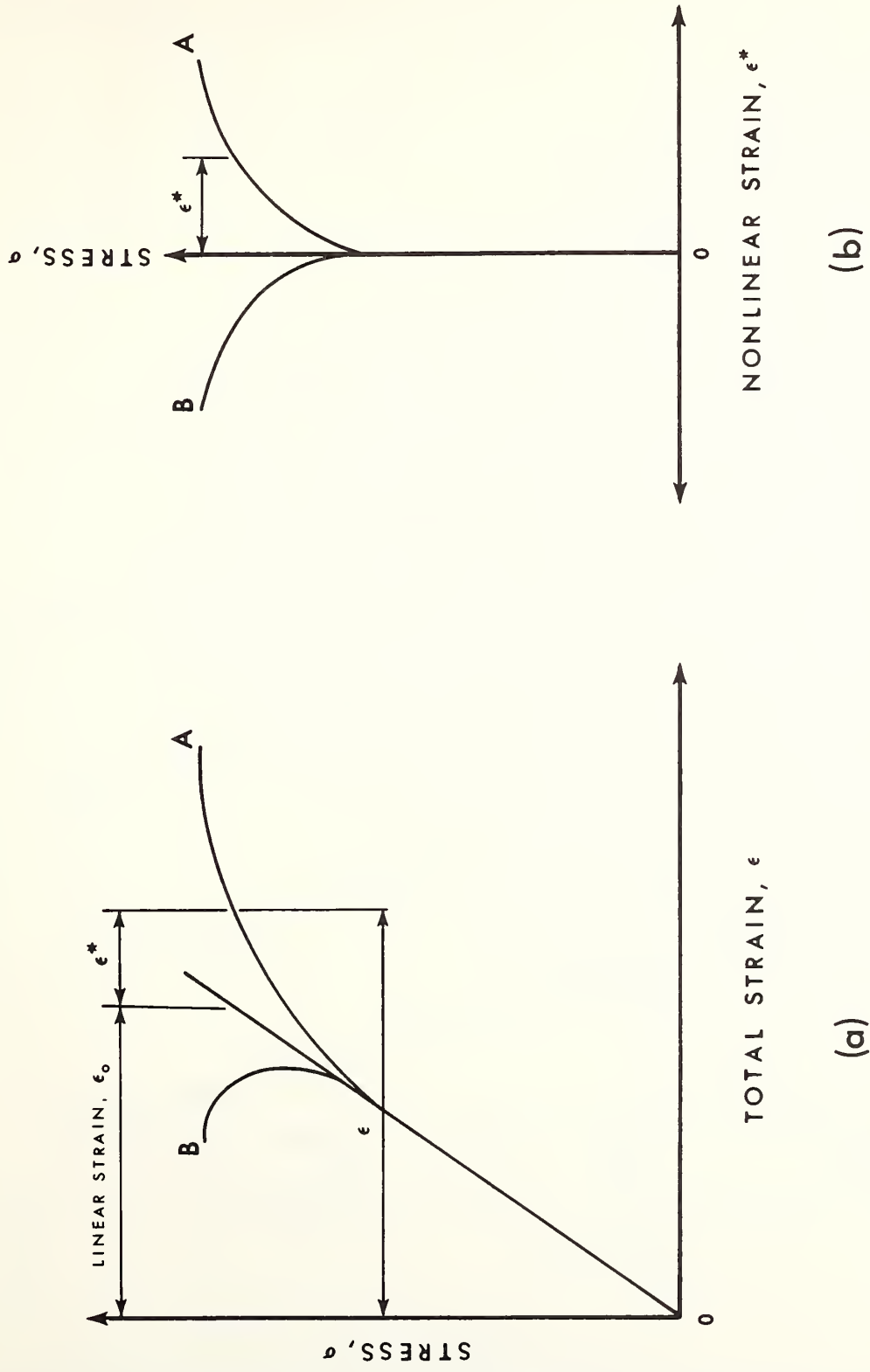


Figure 3. Typical stress-strain relationship observed in mechanical testing.

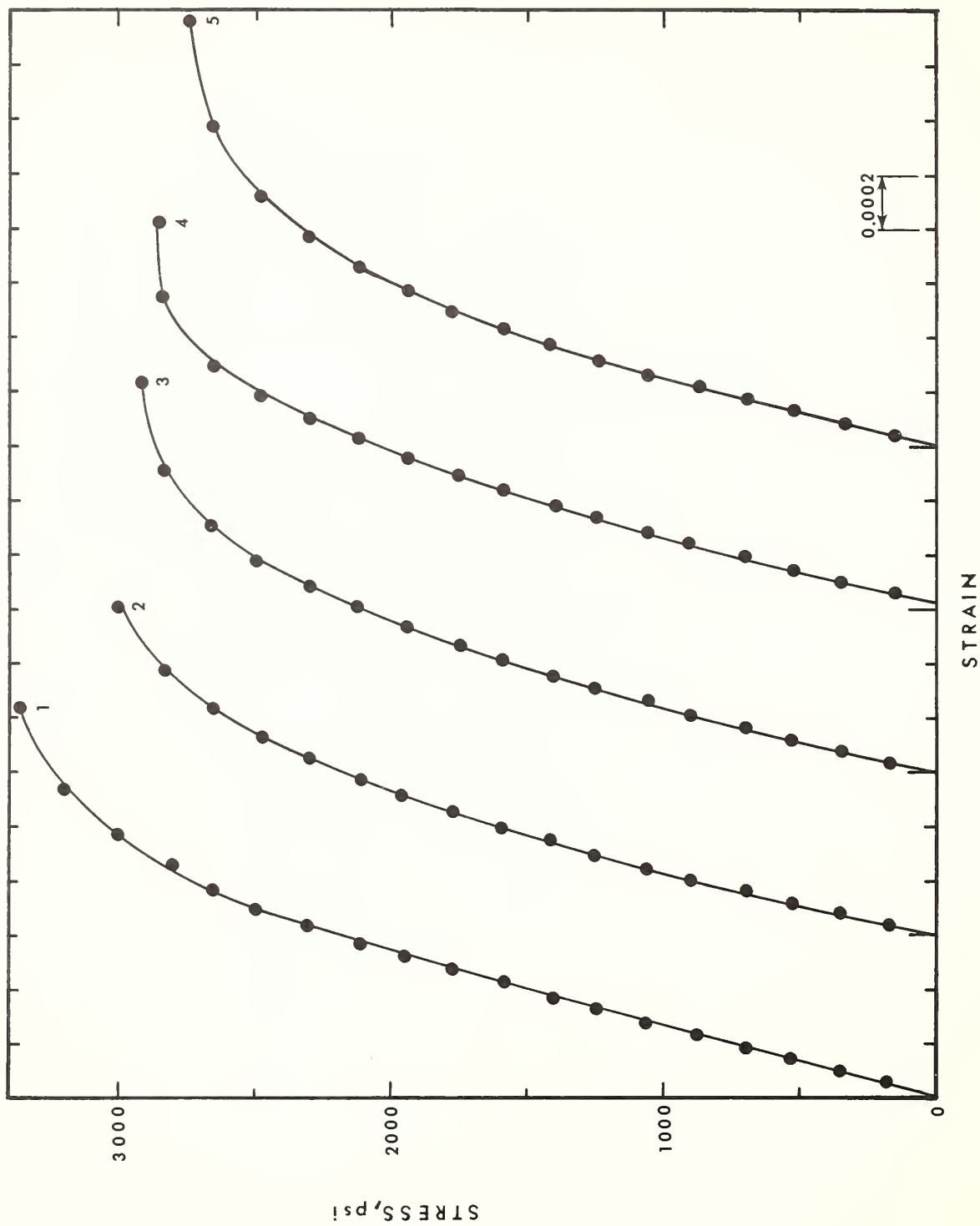


Figure 4. Results of compression tests of 6 by 12 in. concrete cylinders.

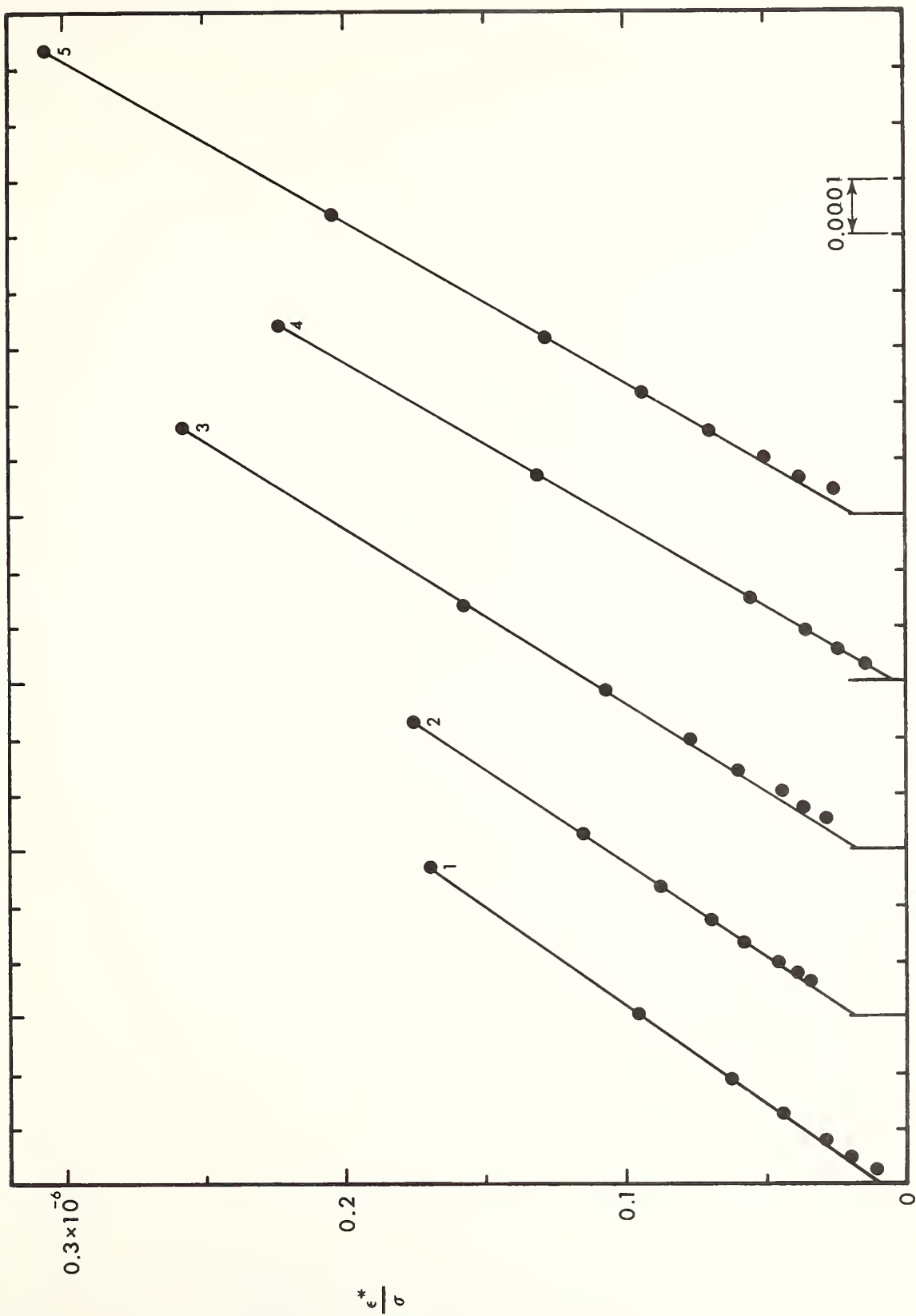


Figure 5. Graphical solution for predicting compressive strength.

NBS TECHNICAL PUBLICATIONS

PERIODICALS

JOURNAL OF RESEARCH reports National Bureau of Standards research and development in physics, mathematics, chemistry, and engineering. Comprehensive scientific papers give complete details of the work, including laboratory data, experimental procedures, and theoretical and mathematical analyses. Illustrated with photographs, drawings, and charts.

Published in three sections, available separately:

● Physics and Chemistry

Papers of interest primarily to scientists working in these fields. This section covers a broad range of physical and chemical research, with major emphasis on standards of physical measurement, fundamental constants, and properties of matter. Issued six times a year. Annual subscription: Domestic, \$5.00; foreign, \$6.00*.

● Mathematics and Mathematical Physics

Studies and compilations designed mainly for the mathematician and theoretical physicist. Topics in mathematical statistics, theory of experiment design, numerical analysis, theoretical physics and chemistry, logical design and programming of computers and computer systems. Short numerical tables. Issued quarterly. Annual subscription: Domestic, \$2.25; foreign, \$2.75*.

● Engineering and Instrumentation

Reporting results of interest chiefly to the engineer and the applied scientist. This section includes many of the new developments in instrumentation resulting from the Bureau's work in physical measurement, data processing, and development of test methods. It will also cover some of the work in acoustics, applied mechanics, building research, and cryogenic engineering. Issued quarterly. Annual subscription: Domestic, \$2.75; foreign, \$3.50*.

TECHNICAL NEWS BULLETIN

The best single source of information concerning the Bureau's research, developmental, cooperative and publication activities, this monthly publication is designed for the industry-oriented individual whose daily work involves intimate contact with science and technology—for *engineers, chemists, physicists, research managers, product-development managers, and company executives*. Annual subscription: Domestic, \$1.50; foreign, \$2.25*.

*Difference in price is due to extra cost of foreign mailing.

NONPERIODICALS

Applied Mathematics Series. Mathematical tables, manuals, and studies.

Building Science Series. Research results, test methods, and performance criteria of building materials, components, systems, and structures.

Handbooks. Recommended codes of engineering and industrial practice (including safety codes) developed in cooperation with interested industries, professional organizations, and regulatory bodies.

Miscellaneous Publications. Charts, administrative pamphlets, Annual reports of the Bureau, conference reports, bibliographies, etc.

Monographs. Major contributions to the technical literature on various subjects related to the Bureau's scientific and technical activities.

National Standard Reference Data Series. NSRDS provides quantitative data on the physical and chemical properties of materials, compiled from the world's literature and critically evaluated.

Product Standards. Provide requirements for sizes, types, quality and methods for testing various industrial products. These standards are developed cooperatively with interested Government and industry groups and provide the basis for common understanding of product characteristics for both buyers and sellers. Their use is voluntary.

Technical Notes. This series consists of communications and reports (covering both other agency and NBS-sponsored work) of limited or transitory interest.

CLEARINGHOUSE

The Clearinghouse for Federal Scientific and Technical Information, operated by NBS, supplies unclassified information related to Government-generated science and technology in defense, space, atomic energy, and other national programs. For further information on Clearinghouse services, write:

Clearinghouse
U.S. Department of Commerce
Springfield, Virginia 22151

Order NBS publications from:
Superintendent of Documents
Government Printing Office
Washington, D.C. 20402

PENN STATE UNIVERSITY LIBRARIES
A000071847085
U.S. DEPARTMENT OF COMMERCE

U.S. DEPARTMENT OF COMMERCE
WASHINGTON, D.C. 20230

OFFICIAL BUSINESS
